



NPTEL ONLINE CERTIFICATION COURSES

FINITE ELEMENT METHOD

Biswanath Banerjee & Amit Shaw

Department of Civil Engineering, IIT Kharagpur

Module 09: FEM for 2D Elasticity: Formulation

Lecture 42 : 2D Elasticity and Weak Form

Concepts Covered

- 2D elasticity Theory in brief
- Weak Formulation





**THANK
YOU !**



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Lecture 43 : Weak Form and Matrix formulation

Concepts Covered

- Weak form to matrix formulation
- Stiffness and force matrix: CST Element



**THANK
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Lecture 44 : Weak Form to Matrix Form

Concepts Covered

- Traction Vector Computation





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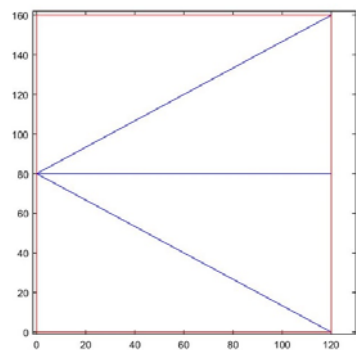
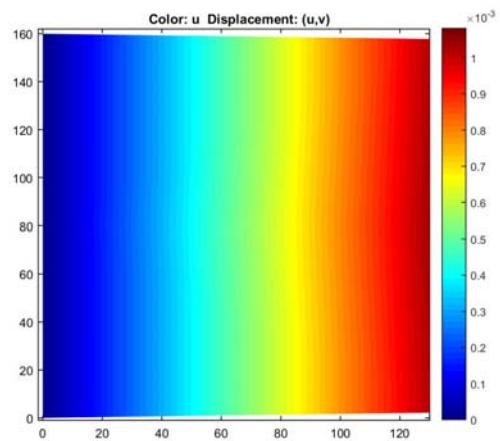
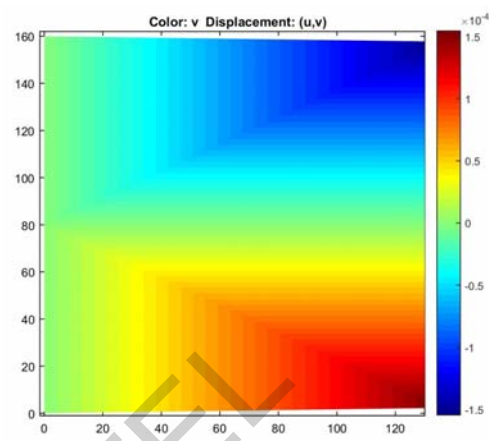
Module 09: FEM for 2D Elasticity: Formulation

Lecture 45 : Problems

Concepts Covered

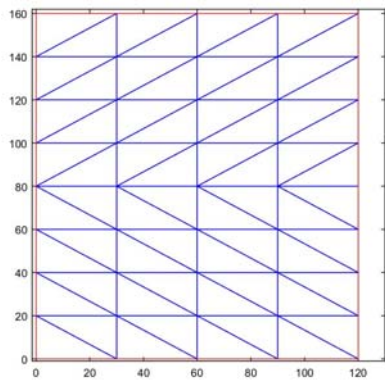
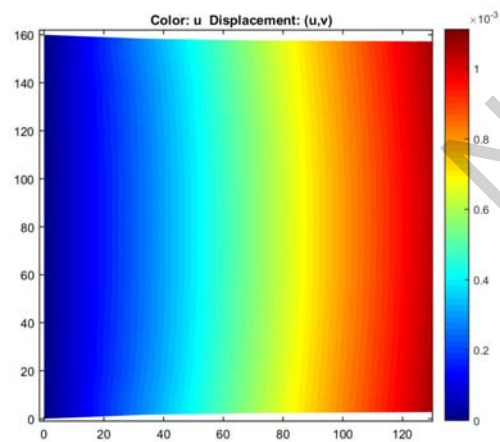
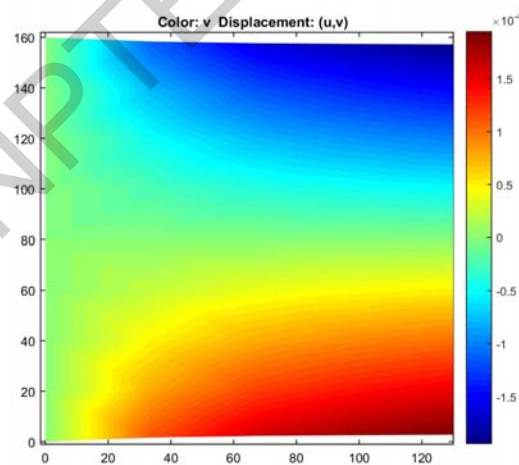
- Plate with uniform axial force





$$u(120,0) = 10.59893 \times 10^{-4}$$

$$v(120,0) = 1.544930 \times 10^{-4}$$

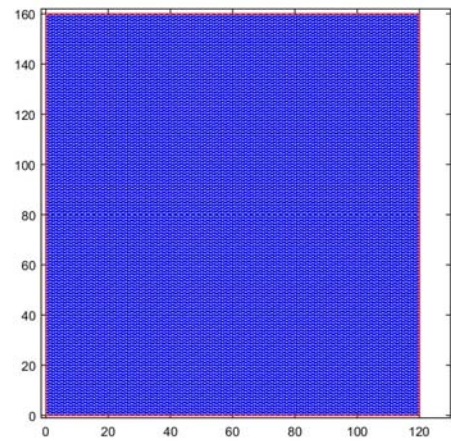
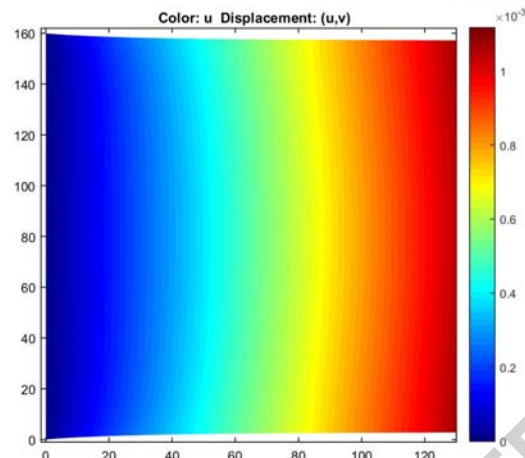
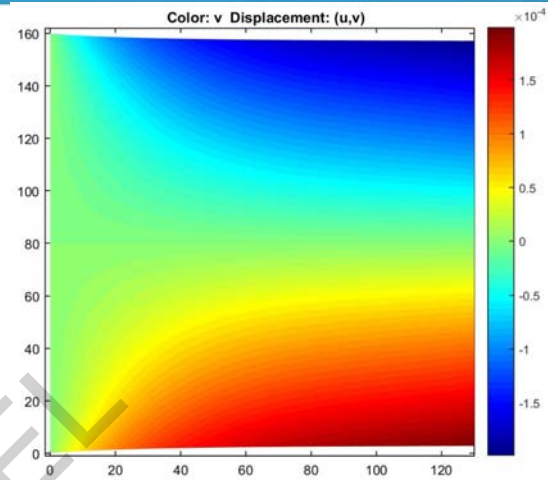
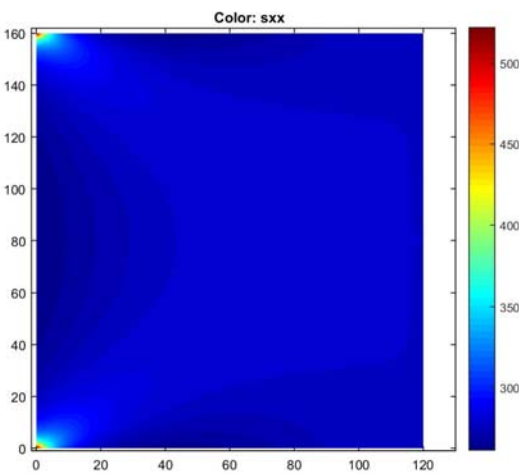
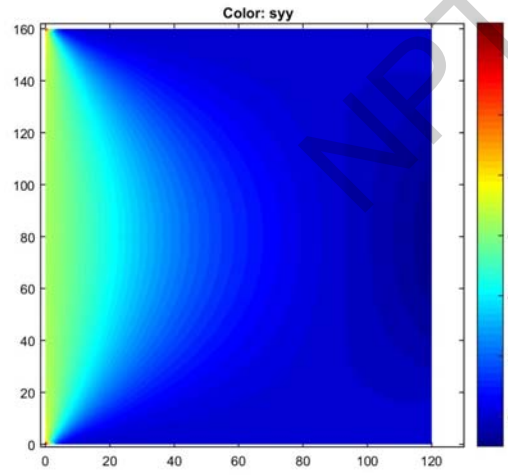
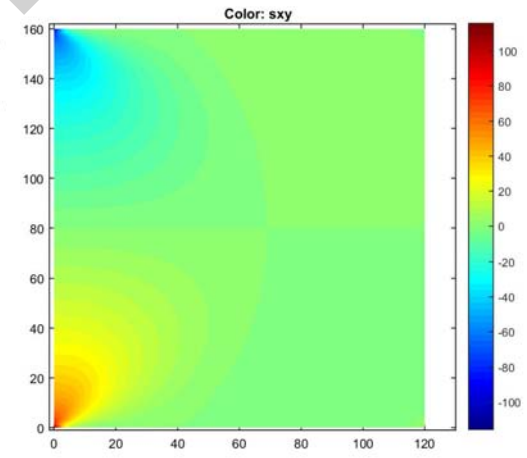


$$u(120,0) = 11.10857 \times 10^{-4}$$

$$v(120,0) = 1.944161 \times 10^{-4}$$

Solution
with fine Mesh

$$u(120,0) = 11.16860 \times 10^{-4}$$
$$v(120,0) = 1.990015 \times 10^{-4}$$





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Lecture 46 : Thermoelastic Problem

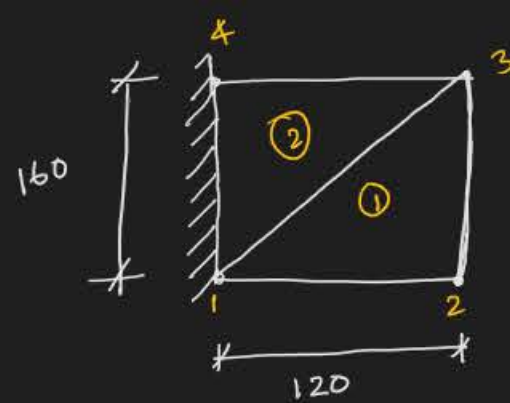
Concepts Covered

- Thermoelastic Problems

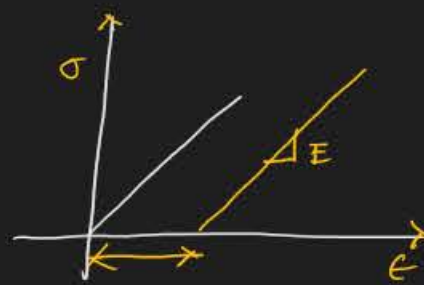
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$$\begin{aligned} E &= 30 \times 10^6 \\ \nu &= 0.25 \\ \alpha &= 1.2 \times 10^{-6} / ^\circ\text{C} \\ T &= 50^\circ\text{C} \end{aligned}$$



$$\begin{aligned} \nabla \cdot \sigma &= 0 \Rightarrow \nabla \cdot (D : \epsilon(u) - D : \epsilon_T) = 0 \\ \sigma \cdot n &= 0 \\ u &= 0 \end{aligned}$$

$$\sigma = D : [\epsilon(u) - \epsilon_T]$$

$$\sigma = D : \epsilon$$

$$\begin{aligned} U &= \frac{1}{2} \int_{\Omega} \sigma : (\epsilon(u) - \epsilon_T) \, d\Omega \\ &= \frac{1}{2} \int_{\Omega} [\epsilon(u) - \epsilon_T] : D : [\epsilon(u) - \epsilon_T] \, d\Omega \end{aligned}$$

$$A^T = \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix}$$

$$[K_e] \{ \Delta \} = \underbrace{\{ f^e \}}_{\parallel} + \underbrace{\{ Q^e \}}_{\parallel \atop 0}$$

$$\{ f^e \} = t \int_{\Omega} [N]^T \begin{Bmatrix} f_x^e \\ f_y^e \end{Bmatrix} dx dy$$

plane stress: -

$$\epsilon_T = \begin{Bmatrix} \epsilon_{Txx} \\ \epsilon_{Tyy} \\ \epsilon_{Txy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{Txx} \\ \epsilon_{Tyy} \\ 0 \end{Bmatrix} = \alpha T \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

plane strain

$$\epsilon_T = (1+\nu) \begin{Bmatrix} \alpha T \\ \alpha T \\ 0 \end{Bmatrix}$$

$$\{ f_T^e \} = t \int_{\Omega} B^T D \epsilon_T \, dx dy$$

$$\begin{aligned} &= t A B^T D \epsilon_T \\ &= \frac{tA}{2A} \begin{bmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_2 & \beta_2 & \gamma_3 & \beta_3 \end{bmatrix}^T \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \alpha T \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \end{aligned}$$

$$= \frac{Et\alpha T}{2(1-\nu^2)} [B]^T \begin{Bmatrix} 1+\nu \\ 1+\nu \\ 0 \end{Bmatrix}$$

$$= \frac{Et\alpha T}{2(1-\nu)} [B]^T \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$

$$\{ f_T^e \} = \frac{Et\alpha T}{2(1-\nu)} \begin{Bmatrix} \beta_1 \\ \gamma_1 \\ \beta_2 \\ \gamma_2 \\ \beta_3 \\ \gamma_3 \end{Bmatrix}_{6 \times 1}$$

$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$



**THANK
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